Mathcad File Showing Progression of Marching Grid Method

Objective Function to be Minimized:

\[ f(x, y) := \frac{x - y}{(x^2 + 5)(y^2 + 5)} + \frac{y^2}{20000} \]

Center of grid is in purple
Minimum point on grid is in yellow

Iteration 1

\( f(3, 6) = -3.426 \times 10^{-3} \)  \( f(4, 6) = -5.229 \times 10^{-4} \)  \( f(5, 6) = 9.87 \times 10^{-4} \)
\( f(3, 5) = -3.512 \times 10^{-3} \)  \( f(4, 5) = -3.373 \times 10^{-4} \)  \( f(5, 5) = 1.25 \times 10^{-3} \)
\( f(3, 4) = -2.601 \times 10^{-3} \)  \( f(4, 4) = 8 \times 10^{-4} \)  \( f(5, 4) = 2.387 \times 10^{-3} \)

Iteration 2

\( f(2, 6) = -9.04 \times 10^{-3} \)  \( f(3, 6) = -3.426 \times 10^{-3} \)  \( f(4, 6) = -5.229 \times 10^{-4} \)
\( f(2, 5) = -9.861 \times 10^{-3} \)  \( f(3, 5) = -3.512 \times 10^{-3} \)  \( f(4, 5) = -3.373 \times 10^{-4} \)
\( f(2, 4) = -9.782 \times 10^{-3} \)  \( f(3, 4) = -2.601 \times 10^{-3} \)  \( f(4, 4) = 8 \times 10^{-4} \)

Iteration 3

\( f(1, 6) = -0.019 \)  \( f(2, 6) = -9.04 \times 10^{-3} \)  \( f(3, 6) = -3.426 \times 10^{-3} \)
\( f(1, 5) = -0.021 \)  \( f(2, 5) = -9.861 \times 10^{-3} \)  \( f(3, 5) = -3.512 \times 10^{-3} \)
\( f(1, 4) = -0.023 \)  \( f(2, 4) = -9.782 \times 10^{-3} \)  \( f(3, 4) = -2.601 \times 10^{-3} \)

Iteration 4

\( f(0, 5) = -0.032 \)  \( f(1, 5) = -0.021 \)  \( f(2, 5) = -9.861 \times 10^{-3} \)
\( f(0, 4) = -0.037 \)  \( f(1, 4) = -0.023 \)  \( f(2, 4) = -9.782 \times 10^{-3} \)
\( f(0, 3) = -0.042 \)  \( f(1, 3) = -0.023 \)  \( f(2, 3) = -7.487 \times 10^{-3} \)
Iteration 5

\[ f(−1,4) = −0.039 \quad f(0,4) = −0.037 \quad f(1,4) = −0.023 \]
\[ f(−1,3) = −0.047 \quad f(0,3) = −0.042 \quad f(1,3) = −0.023 \]
\[ f(−1,2) = −0.055 \quad f(0,2) = −0.044 \quad f(1,2) = −0.018 \]

Iteration 6

\[ f(−2,3) = −0.039 \quad f(−1,3) = −0.047 \quad f(0,3) = −0.042 \]
\[ f(−2,2) = −0.049 \quad f(−1,2) = −0.0554 \quad f(0,2) = −0.044 \]
\[ f(−2,1) = −0.055506 \quad f(−1,1) = −0.055506 \quad f(0,1) = −0.033 \]

Iteration 7

\[ f(−2,2) = −0.049 \quad f(−1,2) = −0.0554 \quad f(0,2) = −0.044 \]
\[ f(−2,1) = −0.055506 \quad f(−1,1) = −0.055506 \quad f(0,1) = −0.033 \]
\[ f(−2,0) = −0.044444 \quad f(−1,0) = −0.033333 \quad f(0,0) = 0 \]

Location of minumum hasn’t changed, so the grid size is reduced. Now, \( \Delta x = \Delta y = 0.5 \)

Iteration 8

\[ f(−1.5,1.5) = −0.05696 \quad f(−1,1.5) = −0.057359 \quad f(−0.5,1.5) = −0.052 \]
\[ f(−1.5, 1) = −0.057421 \quad f(−1, 1) = −0.055506 \quad f(−0.5,1) = −0.048 \]
\[ f(−1.5,0.5) = −0.052533 \quad f(−1,0.5) = −0.047607 \quad f(−0.5,0.5) = −0.036 \]

Iteration 9

\[ f(−2,1.5) = −0.05353 \quad f(−1.5,1.5) = −0.056962 \quad f(−1,1.5) = −0.057359 \]
\[ f(−2,1) = −0.055506 \quad f(−1.5, 1) = −0.057421 \quad f(−1,1) = −0.055506 \]
\[ f(−2,0.5) = −0.052898 \quad f(−1.5,0.5) = −0.052533 \quad f(−1,0.5) = −0.047607 \]
Location of minimum hasn't changed, so the grid size is reduced. Now, $\Delta x = \Delta y = 0.25$

Iteration 10

\[
\begin{align*}
  f(-1.75, 1.25) &= -0.05662 & f(-1.5, 1.25) &= -0.057722 & f(-1.25, 1.25) &= -0.057972 \\
  f(-1.75, 1) &= -0.056798 & f(-1.5, 1) &= -0.057421 & f(-1.25, 1) &= -0.057093 \\
  f(-1.75, 0.75) &= -0.055716 & f(-1.5, 0.75) &= -0.055764 & f(-1.25, 0.75) &= -0.054761
\end{align*}
\]

Location of minimum hasn't changed, so the grid size is reduced. Now, $\Delta x = \Delta y = 0.125$

The process continues using the same algorithm until $\Delta x$ and $\Delta y$ are small enough that the solution can be assumed to have converged.