Given: A collar slides on a vertical rod and is fixed to a spring as shown. The rest position of the collar is the one that minimizes total energy

\[ E(x) = \frac{1}{2} K (2 + x^2 - 2 \sqrt{1 + x^2}) + mgx \]

Note: \( g = 9.81 \text{ m/sec}^2 \)

Find: a) Rest position analytically. Set \( \frac{dE}{dx} = 0 \) and solve for \( x \). There is no exact solution, so use the solve feature on your calculator or iterate to find \( x \). Find \( x \) to 3 sig.fig.

b) Using sequential quadratic approximation. \( x_0 = 1, \Delta x_0 = 1 \) do two iterations. For second iteration, \( \Delta x = 0.5 \)

c) Use binary search where \( x_0 = 0, \Delta x = 0.6 \). Complete two full iterations - complete the iteration for which \( \Delta x = 0.15 \). Do not do the iteration for which \( \Delta x = 0.075 \).
② For the method of steepest descent, successive search directions are: (circle one)

   a) parallel
   b) acute
   c) perpendicular
   d) obtuse
   e) none of the above

③ \( f(x,y) = -\left[ \frac{1}{4} (x^2+y^2) (y-x^2) \right] + y^2 + x^2 \)

   a) Write out the expression for the gradient, \( \nabla f(x,y) \)

   b) If \( x_0 = 2 \), \( y_0 = 4 \) find the initial search direction, \( s \)
      for the method of steepest descent

   c) Show that \( (0, 0.775) \) is the minimum of \( f \).